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Journal of Geometry and Physics 45 (2003) 431–433

JOURNAL OF GEOMETRY AND PHYSICS

www.elsevier.com/locate/jgp

Brief correspondence

Corrections to "Space-like submanifolds in de Sitter spaces" [J. Geom. Phys. 40 (2002) 370–378]☆

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Received 14 March 2002; received in revised form 22 July 2002

Abstract

In this paper, we corrected some errors in [J. Geom. Phys. 40 (2002) 370]. In [J. Geom. Phys. 40 (2002) 370], the errors in Eqs. (32) and (44) influenced the conclusion of Theorem 3.1, Corollary 3.1 and Theorem 3.2.

MSC: 53C40; 53C50; 53C42

Subj. Class: Differential geometry; General relativity

Keywords: Space-like hypersurface; de Sitter space; Totally umbilical

1. Some errors in [1]

There are some errors in Eqs. (32)–(34) in [1]. The errors in (33) and (34) are little typing errors, but the error at (32) was kept in the later computation. The corrected forms of the three equations are as follows:

$$\frac{1}{2}\sum_{i,j} R_{ijij}(\lambda_i^{n+1}-\lambda_j^{n+1})^2 = ncS_{n+1} - n^2H^2c + \left(\sum_{i,a}\lambda_i^a\lambda_i^{n+1}\right)^2 - nH\sum_i(\lambda_i^{n+1})^3,$$

$$\sum_i u_i^{n+1} = 0,\ |z|^2 = S_{n+1} - nH^2, \qquad \sum_i(\lambda_i^{n+1})^3 = \sum_i(u_i^{n+1})^3 + 3H|Z|^2 + nH^3.$$

☆ PII of original article S0393-0440(01)00042-0.

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PII: S0393-0440(02)00152-3

Now, we rewrite Eq. (35) by using the corrected Eqs. (32)–(34) as follows:

$$\frac{1}{2}\sum_{i,j} R_{ijij}(\lambda_i^{n+1} - \lambda_j^{n+1})^2 = |Z|^2(nc - 3nH^2) - n^2H^4 + \left(\sum_{i,a}\lambda_i^a\lambda_i^{n+1}\right)^2 - nH\sum_i (u_i^{n+1})^3.$$

Eqs. (36)–(38) are also wrong because they came from the above equations.

There is also an error in Eq. (44). We correct it as follows:

$$nc - nH^2 - n(n-2)H\sqrt{\frac{\bar{S}_{n+1}}{n(n-1)}} + \bar{S}_{n+1} = nc + Q(u,t) = nc + \frac{n(\bar{u}^2 - \bar{t}^2)}{2\sqrt{n-1}} = nc + \frac{n(-\bar{u}^2 - \bar{t}^2)}{2\sqrt{n-1}} + \frac{n\bar{u}^2}{\sqrt{n-1}} \geq nc - \frac{nS_{n+1}}{2\sqrt{n-1}}.$$

These errors have great influence to Theorem 3.1, Corollary 3.1 and Theorem 3.2 in [1].

2. The corrections of results in [1]

It is difficult to obtain the original results in [1] after correcting the equations. However, if we consider space-like hypersurfaces instead of space-like submanifolds in de Sitter space, the original equations in [1] are correct except (44) ((44) should be the corrected form as above). From the proof in [1], it is easy to get following:

Theorem 2.1. *Let M^n be an n-dimensional compact space-like hypersurface in an $(n+1)$-dimensional de Sitter space $M_1^{n+1}(c)$. If*

$$|\nabla h|^2 \geq n^2|\nabla H|^2 \tag{39}$$

and

$$H^2 < 4\frac{(n-1)c}{n^2}, \tag{40}$$

then $S = nH^2$ and M^n is an umbilical hypersurface.

Corollary 2.1. *Let M^n be an n-dimensional compact space-like hypersurface with constant scalar curvature R in an $(n+1)$-dimensional de Sitter space $M_1^{n+1}(c)$. If $R - c \leq 0$ and*

$$H^2 < 4\frac{(n-1)c}{n^2}, \tag{40}$$

then $S = nH^2$ and M^n is an umbilical hypersurface.

Theorem 2.2. *Let* M^n $(n \geq 3)$ *be an n-dimensional compact space-like hypersurface in the de Sitter space* $M_1^{n+1}(c)$. *If*

$$|\nabla h|^2 \geq n^2 |\nabla H|^2 \tag{39}$$

and

$$S < 2\sqrt{n-1}c, \tag{46}$$

then $S = nH^2$ *and* M^n *is an umbilical hypersurface. Especially, if* M^n *has constant mean curvature and* $S < 2\sqrt{n-1}c$, M^n *is an umbilical hypersurface.*

After consideration of the example in [2], the statements of Theorems 2.1 and 2.2 proposed in the present note seem to be the best formulations that can be achieved.

Acknowledgements

This work was supported by National Natural Science Foundation of China (Grant no. 19971081).

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